

Two-Dimensional Motion Formulas from Physics(Chapter 3)

The formula for displacement or position with constant acceleration “ \mathbf{a} ”, initial velocity \mathbf{v}_0 , time “ \mathbf{t} ”, and initial position x_0 in one dimension is given below:

$x = x_0 + v_0 t + \frac{1}{2} a t^2$. This equation needs to be generalized for two-dimensional motion. In two dimensions, the displacement has a horizontal component and a vertical component as follows: $x = x_0 + v_{x0} t + \frac{1}{2} a_x t^2$ for the x-component and $y = y_0 + v_{y0} t + \frac{1}{2} a_y t^2$ for the y-component. In two dimensions, the velocity has a horizontal component and a vertical component as follows: $v_x = v_{x0} + a_x t$ for the x-component and $v_y = v_{y0} + a_y t$ for the y-component. v_{x0} = the initial velocity for the x-component; and v_{y0} = the initial velocity for the y-component. a_x = the constant acceleration for the x-component; and a_y = the constant acceleration for the y-component.

When dealing with projectile motion, a projectile is fired at an angle θ from the ground with initial velocity \vec{v} . The below figure shows the trigonometric relationship of the components of the initial velocity \vec{v} :

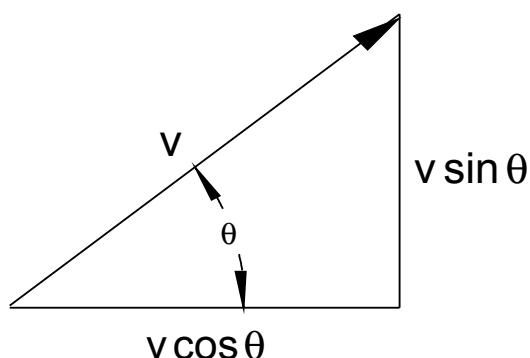


Figure 1: The initial velocity \vec{v} , a vector quantity, is pointing in the direction of the arrow. v = the magnitude of the velocity vector. The horizontal component = $v_{x0} = v \cos \theta$ and the vertical component = $v_{y0} = v \sin \theta$.

The magnitude of the initial velocity vector \vec{v} is v as shown above. In the horizontal direction, acceleration is assumed to be zero and the acceleration is assumed to be $g = 9.8 \text{ m/s}^2$. Hence, $x = x_0 + v_{x0} t$, $v_x = v_{x0}$, $v_y = v_{y0} - gt$, and $y = y_0 + v_{y0} t - \frac{1}{2} gt^2$ are the equations that we will use to solve projectile motion. Usually, the starting point (x_0, y_0) is assumed to be $(0, 0)$. Thus, $x = v_{x0} t$, $v_x = v_{x0}$, $v_y = v_{y0} - gt$, and $y = v_{y0} t - \frac{1}{2} gt^2$ are the equations we will use. Substituting $x = v_{x0} t$ into $y = v_{y0} t - \frac{1}{2} gt^2$

we have $y = \left[\frac{v_{y0}}{v_{x0}} \right] x - \frac{1}{2} g \left[\frac{v_{y0} x}{v_{x0}} \right]^2 \Rightarrow y = \left[\frac{v_{y0}}{v_{x0}} \right] x - \left[\frac{g}{2v_{x0}^2} \right] x^2$. In the case where

$v_{x0} = v \cos \theta$ and $v_{y0} = v \sin \theta$, $y = (\tan \theta)x - \left[\frac{g}{2v_0^2 \cos^2 \theta} \right] x^2$. Using the previous formula, (set $y = 0$ and solve for $x = R_e$) we have the formula for the horizontal range R_e on Earth is $R_e = \frac{v_0^2 \sin 2\theta_0}{g}$. The maximum height is $h_e = \frac{v_0^2 \sin^2 \theta_0}{g}$.

Another kind of two-dimensional motion is circular motion of an object where the magnitude v of the velocity \vec{v} of the object is constant. Just because the velocity \vec{v} is pointing in one direction does not mean that the acceleration \vec{a} is pointing in the same direction. The acceleration \vec{a} is called the centripetal acceleration. The below figure shows the relationship between velocity \vec{v} and centripetal acceleration \vec{a} of an object in circular motion:

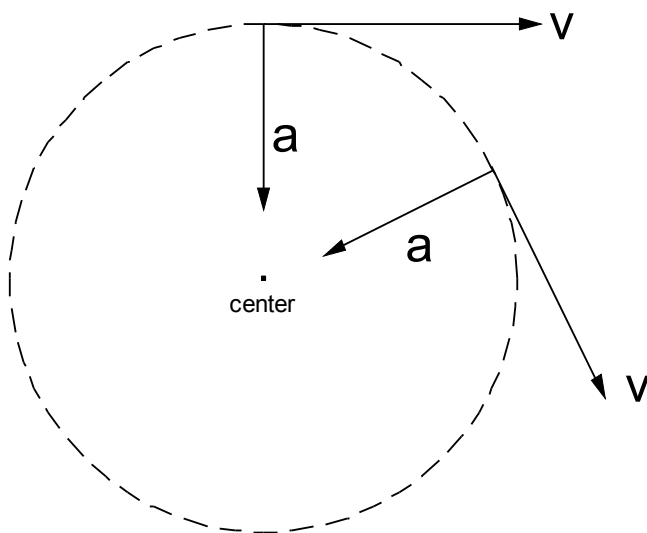


Figure 2: The above figure shows that the velocity and the centripetal acceleration vectors are perpendicular.

The magnitude of the centripetal acceleration $a = \frac{v^2}{r}$ where r = the radius of the circle and v = the magnitude v of the velocity \vec{v} (speed).

Formulas that are useful in calculating the velocity's magnitude v (speed) is $F = \frac{1}{T}$ where F = the frequency in revolutions per second and T = the period = the amount of time to complete one revolution of the circle. Multiplying the frequency by 2π gives the angular speed $A = \frac{2\pi}{T}$ and multiplying A by the radius r gives the speed $v = \frac{2\pi r}{T}$.