

SIMPSON'S RULE AND THE TRAPEZOIDAL RULE

Two Rules for Approximating Integrals:

$$1. \int_a^b f(x)dx = \frac{b-a}{3n} [f(x_0) + 4f(x_1) + 2f(x_2) + 4f(x_3) + \dots + 2f(x_{n-2}) + 4f(x_{n-1}) + f(x_n)] = \text{Simpson's Rule}$$

$$2. \int_a^b f(x)dx = \frac{b-a}{2n} [f(x_0) + 2f(x_1) + 2f(x_2) + \dots + 2f(x_{n-2}) + f(x_n)] = \text{Trapezoidal Rule}$$

Note that the function $f(x)$ is defined on the interval $[a,b]$. The interval $[a,b]$ is broken into n pieces of equal length. Thus, each piece has length $\frac{b-a}{n}$.

Example: Approximate $\int_1^2 (1/x)dx$ using Simpson's Rule and the Trapezoidal Rule where $n = 4$ ($n =$ the number of pieces).

First split the interval $[1,2]$ ($a = 1 =$ lower limit of integration; $b = 2 =$ upper limit of integration) into 4 pieces of equal length. Each piece has length $(2-1)/4 = 1/4$.

Note $x_0 = 1$, $x_1 = x_0 + 1/4 = 5/4$, $x_2 = x_1 + 1/4 = 3/2$, $x_3 = x_2 + 1/4 = 7/4$, $x_4 = x_3 + 1/4 = 2$.

Thus, $f(x_0) = 1$, $f(x_1) = 4/5$, $f(x_2) = 2/3$, $f(x_3) = 4/7$, $f(x_4) = 1/2$.

By the trapezoidal rule: $\int_1^2 (1/x)dx \approx 1/8(1 + 2(4/5) + 2(2/3) + 2(4/7) + 1/2) =$

$$1/8(1 + 8/5 + 4/3 + 8/7 + 1/2) = (1/8 + 1/5 + 1/6 + 1/7 + 1/16) = .7$$

By Simpson's rule: $\int_1^2 (1/x)dx \approx 1/12(1 + 4(4/5) + 2(2/3) + 4(4/7) + 1/2) =$

$$1/12(1 + 16/5 + 4/3 + 16/7 + 1/2) = (1/12 + 4/15 + 1/9 + 4/21 + 1/24) = .692$$

The true value is equal to $\int_1^2 (1/x)dx = \ln(x)|_1^2 = (\ln 2 - \ln 1) = \ln 2 = .693$