

## One-Dimensional Motion Formulas from Physics(Chapter 2)

The formula for displacement or position with constant acceleration “**a**”, initial velocity **v**<sub>0</sub>, time “**t**”, and initial position **x**<sub>0</sub> in one dimension is given below:

$x = x_0 + v_0 t + \frac{1}{2} a t^2$ . This equation is good for motion along the x-axis, any horizontal surface, or vertical(*up and down*) motion. From calculus, velocity is the derivative of position; i.e.,  $v = \frac{dx}{dt} = v_0 + at$ .

When dealing with vertical motion, we use the acceleration of the Earth which is **9.8 m/s<sup>2</sup>** where **m** = meters and **s** = time in seconds. We use the symbol “**-g**” instead of “**a**” in the above equation because objects eventually fall to the ground(*lose altitude*) when thrown upward. The above equations become  $x = x_0 + v_0 t - \frac{1}{2} g t^2$  and  $v = \frac{dx}{dt} = v_0 - gt$ . The below example deals with vertical motion:

**Example 1:** A rock is thrown upward with an initial velocity of **10 m/s**. Using the acceleration of gravity, **a)** How long will it take the rock to reach its maximum height? **b)** How high will the rock go up? **c)** How long will the rock take to come back down to the ground?

Solution: **a)** When the rock reaches its maximum height, the velocity of the rock  $v = 0$ ; hence,  $0 = v_0 - gt \Rightarrow v_0 = gt \Rightarrow 10 = 9.8t \Rightarrow t = \frac{10}{9.8} = \underline{\underline{1.02 \text{ sec}}}$ .

**b)** Next, we can also assume that the initial position  $x_0 = 0$ . Since  $x_0 = 0$  and the initial velocity  $v_0 = 10 \text{ m/s}$ ,  $x = 10t - \frac{1}{2} g t^2$ . Because the rock reaches its maximum height in 1.02 sec, we can substitute  $t = 1.02$  into the above equation and get the result below:

$$t := 1.02 \quad x := 10 \cdot t - 4.9 \cdot t^2 \quad x = 5.102 \text{ meters}$$

**c)** When the rock comes back to the ground, **x** will equal zero and **t** will not equal zero. Hence,  $0 = 10t - 4.9t^2 = t(10 - 4.9t) \Rightarrow 4.9t = 10 \Rightarrow t = \underline{\underline{2.04 \text{ sec}}}$ .

Since  $v = v_0 + at$ ,  $t = \frac{v - v_0}{a}$ . Substituting that into  $x = x_0 + v_0 t + \frac{1}{2} a t^2$ , we get

$$x = x_0 + v_0 \left[ \frac{v - v_0}{a} \right] + \frac{1}{2} a \left[ \frac{v - v_0}{a} \right]^2 \Rightarrow (x - x_0) = \frac{v v_0 - v_0^2}{a} + \frac{v^2 - 2v v_0 + v_0^2}{2a}$$

$$\Rightarrow (x - x_0) = \frac{2v v_0 - 2v_0^2}{2a} + \frac{v^2 - 2v v_0 + v_0^2}{2a} \Rightarrow (x - x_0) = \underline{\underline{\frac{v^2 - v_0^2}{2a}}}$$

The above formula is used when time is not given, not needed, or when we want to establish a direct relationship between position and velocity. For vertical motion,

substitute “-g” for “a”. The above formula could have been used to solve part b)

above. Notice that  $x = \frac{-v_0^2}{-2g} = \frac{100}{19.6} = \underline{\underline{5.102 \text{ meters}}}$ .

The next set of formulas is used when we want to figure out average velocity. One way to get the average velocity is to add the initial velocity  $v_0$  and the final

velocity  $v$  and average them; i.e.,  $\bar{v} = \frac{v + v_0}{2}$ . We can also get the average velocity

by subtracting the initial position  $x_0$  from  $x$  and dividing by “t”; i.e.,  $\bar{v} = \frac{x - x_0}{t}$ .

Substituting, we have  $\frac{v + v_0}{2} = \frac{x - x_0}{t}$ . These formulas are useful when we are given

a graph of time versus position. Solving for  $x$  we have  $x = x_0 + \frac{(v + v_0)t}{2}$ . The

previous equation describes position as a function of velocity and time. The below example illustrates the use of the previous equation:

**Example 2:** A particle, moving on the x-axis starting at the origin with a speed of 5 m/s, increased its speed to 10 m/s in 20 seconds. How far from the origin did the particle go?

Solution:  $x_0 = 0$  since the particle is starting at the origin.  $v_0 = 5$  m/s and  $v = 10$  m/s.

Substituting, we have  $x = \frac{(10 + 5)20}{2} = 150$  meters.