(Forces and Newton's Laws):

Newton's Laws:

Isaac Newton discovered three laws of motion for forces:

1. If the net force on an object is zero, then an object at rest will stay at rest, and an object moving at a constant velocity will continue to move at a constant velocity.

2. $\sum F = ma$; i.e., the sum of the forces F acting on an object is directly proportional to the acceleration **a** of the object. Force is a vector quantity and forces are added vectorially.

3. When an object exerts a force $\mathbf{F}_{12}$ on a second object and the net force is zero, the force $\mathbf{F}_{21}$ the second body exerts on the first object is equal in magnitude to $\mathbf{F}_{12}$ but opposite in sign and direction. $\mathbf{F}_{21} = -\mathbf{F}_{12}$.

Common Force Systems to Consider when Solving Problems:

1. **Normal & Gravitational Forces**:

One force to consider when solving problems is the **normal force** F_N. When an object lies on a surface, the normal force F_N is the force that is perpendicular to that surface. An illustration is shown below:

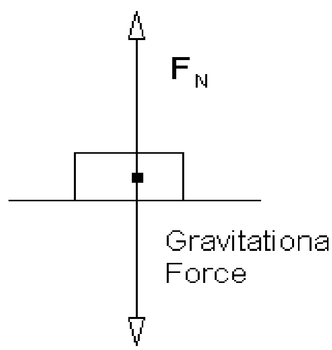
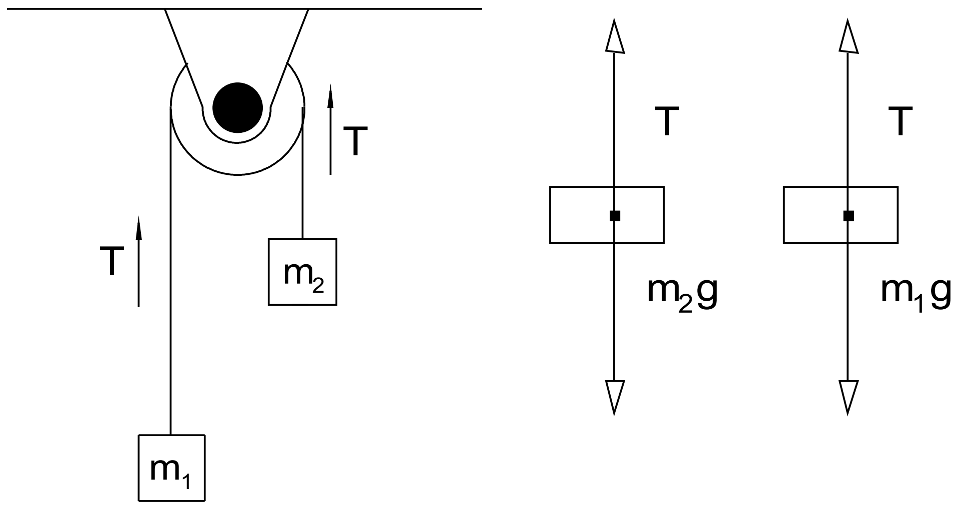


Figure 1: The normal force is shown above. It opposes the gravitational force.

Take a look at figure 1. The gravitational force opposes the normal force F_N and points downward. By Newton's Second law, the gravitational force = **ma** where **m** = the mass of the object and **a** = the acceleration of gravity. The acceleration of gravity = **9.8** m/s^2. The symbol that is used for the gravitational acceleration is **g**. The object is not moving; hence, by Newton's First Law, the net force of the above system is zero. By Newton's Third Law, $F_N - mg = 0 \Rightarrow F_N = mg$. The negative sign for the gravitational force is used here because it is pointing downward.

2. <u>Tension in a Pulley</u>:

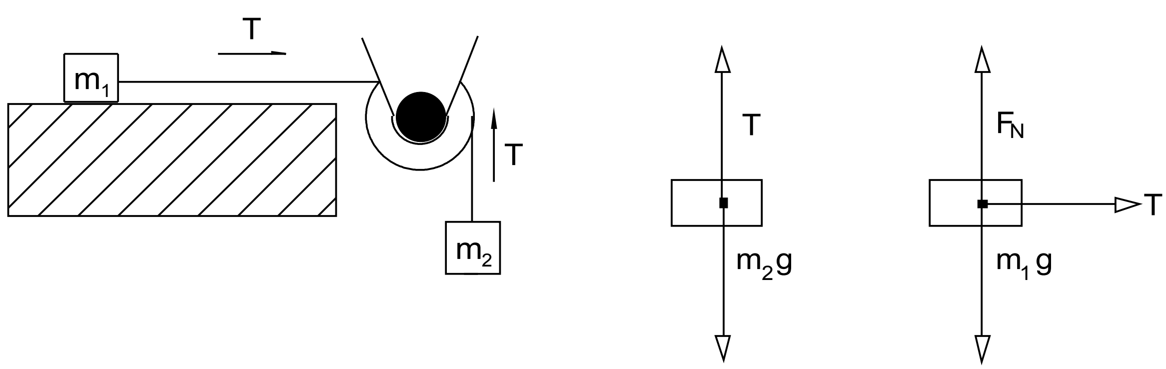
The next force to consider is tension **T**. For a pulley, the tension force points toward the pulley as shown below:



<u>Figure 2</u>: Here is the diagram that illustrates the tension force **T** and the free body diagram for each of the forces on the two masses above.

For pulleys, the masses have a common magnitude of acceleration **a**. From intuition, it appears that mass m_2 is going up with acceleration **a**, and mass m_1 is going down with acceleration **–a**. Hence, you have two equations: $T - m_1 g = -m_1 a$ and $T - m_2 g = m_2 a$ by Newton's Second Law. There are only vertical components of acceleration and force.

We can have horizontal pulley systems as shown below(no friction here):

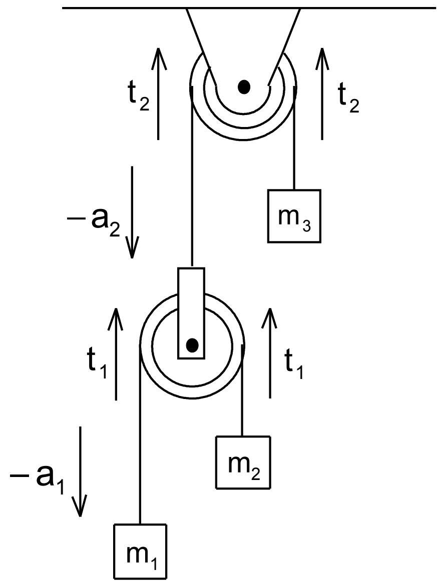


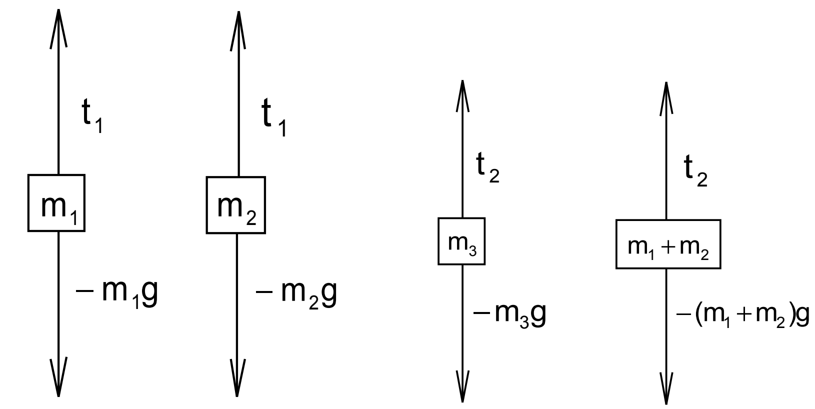
<u>Figure 3</u>: Here is a horizontal pulley system together with its free body diagram.

Adding the vertical components of force, we have $T - m_2 g = -m_2 a$ and $F_N - m_1 g = 0$. Adding the horizontal components of force, we have $Tm_1 = am_1$

There are many variants of pulley systems. The pulley systems above are not the only ones. Many pulley systems need to be handled on a case-by case basis. However, Newton's laws need to be applied and a free body diagram needs to be drawn. To further illustrate the pulley and tension concept, we illustrate the 2-pulley system below:

Equations for a 2-Pulley System:





$$t_1 - m_1 g = (-a_1 - a_2)m_1 \qquad t_2 - m_3 g = a_2 m_3$$
$$t_1 - m_2 g = (a_1 - a_2)m_2 \qquad t_2 - (m_1 + m_2)g = -a_2(m_1 + m_2)$$

Figure 4: This is the two-pulley system that we are analyzing. The derived equations are shown above.

The total acceleration for m_1 is $-(a_1+a_2)$ and the total acceleration for m_2 is $(a_1 - a_2)$. We are assuming that the pulleys are massless or the mass of the pulleys are included in the masses above. t_1 and t_2 are the tensions in the lower pulley and the upper pulley, respectively. WLOG(*without loss of generality*), $m_1 \le m_2$, and $m_1 + m_2 \ge m_3$. The above system can be generalized for three or more pulleys.

3. Forces on an Incline:

The below system shows how to apply Newton's laws to a mass on an incline:

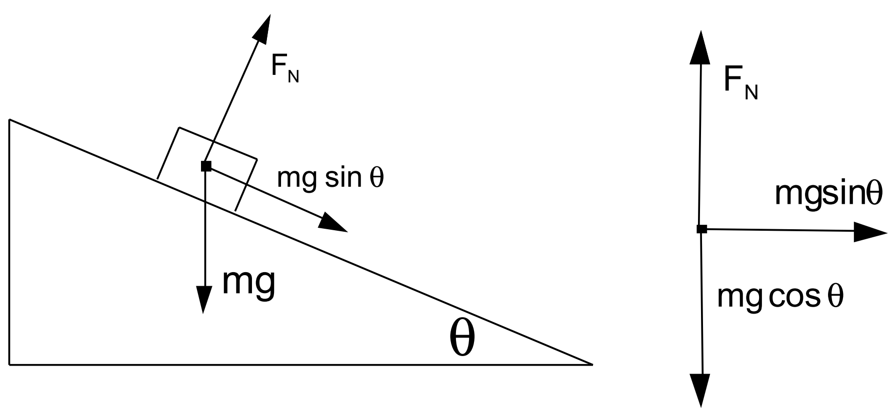


Figure 5: The free body diagram for an object of mass m on an incline is shown above.

When dealing with forces on an incline, let the normal force F_N be on the y-axis. $mg \cdot \sin\theta$ will automatically be on the x-axis. Sum the forces accordingly. From geometry, $F_N = mg \cdot \cos\theta$.