DeMoivre's Theorem & Complex Numbers:

DeMoivre's theorem is used to raise complex numbers to various powers. All complex numbers can be written in the form $re^{i\theta}$ where r = magnitude of the complex number and $e^{i\theta} = \cos\theta + i\sin\theta$. θ = angle that the complex number makes in the complex plane.

To raise a complex number to a power "n", simply raise r to the nth power, and multiply the angle θ by "n" in the expression $e^{i\theta} = \cos\theta + i\sin\theta$ to get $r^n e^{ni\theta} = r^n(\cos n\theta + i\sin n\theta)$. "n" can be any real number. For finding nth roots of numbers, note that $\cos\theta = \cos(\theta + 2k\pi)$ and $\sin\theta = \sin(\theta + 2k\pi)$ where k = any integer. Using this fact, we can write $e^{i\theta} = \cos\theta + i\sin\theta$ as $e^{i\theta} = \cos(\theta + 2k\pi) + i\sin(\theta + 2k\pi)$. To take the nth root of $re^{i\theta} = \cos\theta + i\sin\theta$, we take the nth root of r, and divide (θ + 2kπ) by n in $e^{i\theta} = \cos(\theta + 2k\pi) + i\sin(\theta + 2k\pi)$ to get $r^{1/n}e^{i\theta/n} = r^{1/n}[\cos(\theta/n + 2k\pi/n) + i\sin(\theta/n + 2k\pi/n)]$. Run through the values k = 1,2, …,n to get the **n** nth roots of $re^{i\theta}$ = the complex number.

An example is done below: Find the 3 cube roots of $i = \sqrt{-1}$. First find the magnitude of i. The magnitude = r = $\sqrt{0^2 + 1^2}$ = 1 because a = 0 and b = 1, all complex numbers are of the form a + bi, and the magnitude of a + bi = $\sqrt{a^2 + b^2}$. The cube root of r = 1 is 1. The angle θ that $i = \sqrt{-1}$ makes is 90^0 or $\pi/2$. Thus, i = cos($\pi/2$ + 2kπ) + i sin($\pi/2$ + 2k π) Dividing the angle by 3 we get $\sqrt[3]{i}$ = cos($\pi/6$+$2k\pi/3$) + i sin($\pi/6$+$2k\pi/3$). Running through k = 1,2,3 we get cos($5\pi/6$) + i sin($5\pi/6$) = $-\sqrt{3}/2 + i/2$ for k = 1; cos($3\pi/2$) + i sin($3\pi/2$) = $-i$ for k = 2; and cos($13\pi/6$) + i sin($13\pi/6$) = $\sqrt{3}/2 + i/2$ for k = 3.

DeMoivre's theorem is used to find the zeros of the polynomial $x^n - a = 0$ where "a" is a complex number. It can also be used to find some or all of the complex zeros of polynomials that are divisible by $x^n - a$.

DeMoivre's theorem has applications in electrical engineering and physics.